Derivation of the Uncertainty assigned to the Residual Reflection Tracking Parameter with SOL(T) calibrations

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***Abstract* – Knowledge of the residuals of the error box parameters is essential when the ripple method is used for calculating measurement uncertainty. In contrast to residual directivity and residual source match, residual reflection tracking is generally not accessible via a ripple measurement. It is shown that this parameter can be estimated easily, however, when calibration data of the used calibration standards is available. An analysis of existing estimation methods for model-based calibration standards is presented, when standard definitions are used.**

***Index terms* – VNA calibration, ripple method, residual reflection tracking**

I. Introduction

It is one of the weak points of the ripple method that residual reflection tracking cannot be measured in an easy and fool-proof way. Although there have been attempts to derive this quantity by experiment, the proposed method [1] delivers reliable results only with hermaphroditic connectors, where the same calibration standards are used for both ports.

Therefore, for the vast majority of cases, residual reflection tracking has to be estimated in another way. In the past, the influence of residual reflection tracking on the accuracy of reflection parameters was judged to be of minor importance, when the magnitude only was of interest. As an example, the following statement can be found in in chapter 6.2.4.1 of the previous version of the cg-12 guide [2]: *"Usually it is satisfactory to use the manufacturer's value for this contribution, e.g. a relative uncertainty of 0.001 as half interval of a rectangular distribution."* No information is given to the user, whether this also applies to data-based calibration standards.

When measurements are to be performed, which require phase information, several attempts are in use to account for the phase uncertainty of the reflecting standards. To the author's knowledge, at least one accredited laboratory uses the uncertainty determined for residual source match by the ripple method as a measure for residual reflection tracking. With another approach, simply the manufacturer specifications on phase accuracy of the standards are used.

II. Functional relationships of the residual model

The outcome of a one-port measurement can be written as a function of the residuals of the test port and the true (physical) reflection coefficient of the DUT (see e.g. [3]). For any complex-valued reflection coefficient $\boldsymbol{\Gamma}$ of the DUT one gets an error-corrected measurement result, which can be described by

$$\boldsymbol{\Gamma}^{\mathrm{c}} = \boldsymbol{\Gamma} + \boldsymbol{\Delta\Gamma} = \boldsymbol{\delta} + (1+\boldsymbol{\tau})\frac{\boldsymbol{\Gamma}}{1-\boldsymbol{\mu\Gamma}}\,, \tag{1}$$

when drift and random errors are neglected. $\boldsymbol{\Delta\Gamma}$ represents the measurement error, and $\boldsymbol{\delta}, \boldsymbol{\mu}, \boldsymbol{\tau}$ stand for the complex-valued residuals of directivity, source match and reflection tracking, respectively. This same equation holds, of course, also true for the calibration standards that have been used to calibrate the VNA. Then $\boldsymbol{\Gamma}^{\mathrm{c}}$ is the definition of the standard and $\boldsymbol{\Delta\Gamma}$ can be seen as the definition error.

When eq. (1) is solved for $|\boldsymbol{\mu\Gamma}| << 1$ (i.e. the usual case), one gets the well-known approximation

$$\Delta\Gamma \approx \delta + \tau\Gamma + \mu\Gamma^2 . \tag{2}$$

This functional relationship is represented in Fig. 1 for an arbitrary offset short standard and a congruent, i.e. phase-matched offset open standard. For such standards, which can be found in regular SOLT calibration kits, phase difference is approximately 180° irrespective of the absolute phase angles. Residual directivity δ has also been chosen arbitrary in this example.

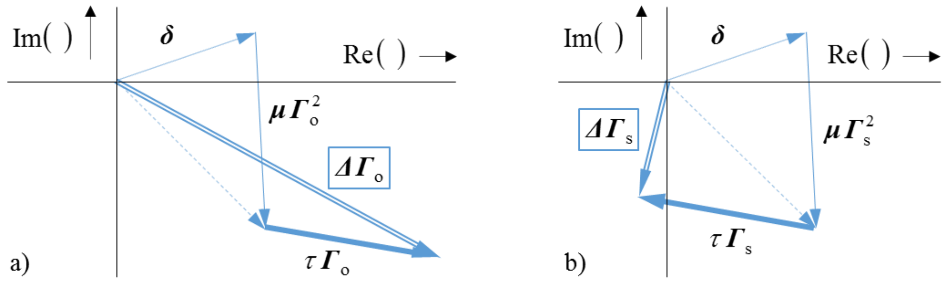

Fig. 1 Definition error $\Delta\Gamma_o$ for an offset open standard (a) and $\Delta\Gamma_s$ for a phase-matched offset short standard (b)

As can easily be seen from Fig. 1, both errors share a common component (dotted vector), which is composed of residual directivity and a term induced by residual source match. Apparently, these portions cancel out when the difference $\Delta\Gamma_o$ -$\Delta\Gamma_s$ is set up. Thus, a relation between the searched quantity (τ) and the definition errors is available, without the need to consider δ and μ additionally.
A detailed mathematical derivation is given in the Appendix, leading to a first-order approximation of

$$\tau \approx \frac{\Delta s + \Delta o}{2} + \frac{\Delta x}{2}\left(\mu\Gamma_s - \frac{\delta}{\Gamma_s}\right) \tag{3}$$

with relative errors

$$\Delta s = \frac{\Delta\Gamma_s}{\Gamma_s},\ \Delta o = \frac{\Delta\Gamma_o}{\Gamma_o} \text{ and congruence parameter } \Delta x = -\frac{\Gamma_o}{\Gamma_s} - 1 . \tag{4}$$

For residual source match and residual directivity one gets accordingly

$$\mu \approx \Gamma_s^{-1} \cdot \left[\frac{\Delta s - \Delta o}{2} - \frac{\Delta\Gamma_m}{\Gamma_s} - \frac{\Delta x}{2}\left(\mu\Gamma_s - \frac{\delta}{\Gamma_s}\right) + \tau\frac{\Gamma_m}{\Gamma_s}\right], \tag{5}$$

$$\delta \approx \Delta\Gamma_m - \Gamma_m\tau \tag{6}$$

The shaded areas indicate contributions that can be neglected for “typical” calibration devices, i.e. a well-matched load and phase-matched high-reflective standards. The latter are most often specified in terms of magnitude and phase errors, which, for small values of relative errors $|\Delta z|$, are related to the real and the imaginary parts as follows:

$$\mathrm{Re}(\Delta z) = \frac{\Delta_{\mathrm{mag}}(\Gamma_z)}{|\Gamma_z|}, \text{ where } \Delta_{\mathrm{mag}}(\Gamma_z) = |\Gamma_z + \Delta\Gamma_z| - |\Delta\Gamma_z| \text{ indicates the magnitude error.} \tag{7}$$

$$\mathrm{Im}(\Delta z) = \Delta_{\mathrm{phi}}(\Gamma_z), \text{ where } \Delta_{\mathrm{phi}}(\Gamma_z) = \arg\left(\frac{\Gamma_z + \Delta\Gamma_z}{\Gamma_z}\right) \text{ indicates the phase error in radians.} \tag{8}$$

Phase errors of the high-reflective standards will thus map onto the imaginary part of τ, and magnitude errors map onto the real part.

III. Implications with using generic manufacturer standard definitions

When calibration standards are used that are characterized by a standardized physical model and according manufacturer specifications, the model is regarded as a reference, whereas the calibration devices are seen prone to errors. Compared to the definitions made in eq. (1), the errors would appear on the right side. The outcome doesn't change, however, if eq. (1) is used with an inverted sign of the device error.

Typically, the errors of the calibration standards within one specific calibration kit can be regarded independent from one another, i.e. the errors being uncorrelated. This stems from the fact that they are mainly induced by the dispersion of geometrical dimensions, surface roughness, material properties and the thickness of electroplating, which differ between different types of calibration devices. When matched sets of calibration standards are assembled, however, e.g. to achieve low residual source match, correlation is enforced.

A standard uncertainty of the imaginary part of $\boldsymbol{\tau}$ can be obtained by couching eq. (3) in terms of uncertainties. When the phase errors of the calibration devices are assumed uncorrelated, and the term with $\boldsymbol{\Delta x}$ is neglected, one gets

$$u^2\left(\mathrm{Im}\left(\hat{\boldsymbol{\tau}}\right)\right) \approx \frac{u_{\mathrm{phi}}^2\left(\hat{\boldsymbol{\Gamma}}_{\mathrm{s}}\right)}{4} + \frac{u_{\mathrm{phi}}^2\left(\hat{\boldsymbol{\Gamma}}_{\mathrm{o}}\right)}{4}, \tag{9}$$

where $u_{\mathrm{phi}}\left(\hat{\boldsymbol{\Gamma}}\right)$ represents a standard uncertainty of the phase angle of the expectation value of a complex-valued quantity $\boldsymbol{\Gamma}$, and $\hat{\boldsymbol{\Gamma}}_{\mathrm{s}} = \boldsymbol{\Gamma}_{\mathrm{s}}^{\mathrm{c}}$, $\hat{\boldsymbol{\Gamma}}_{\mathrm{o}} = \boldsymbol{\Gamma}_{\mathrm{o}}^{\mathrm{c}}$, $\hat{\boldsymbol{\tau}} = 0$ are best estimates.

Example: Phase uncertainties for offset open standards and offset short standards within the Keysight 85054B N calibration kit are specified with ±1.5° and ±1°, respectively. Assuming a uniform distribution between specification limits results in standard uncertainties of 0.87° and 0.58°, respectively, summing up to a total standard uncertainty of 0.005 for the imaginary part of the residual reflection tracking parameter.

Normally, no tolerance is given by the manufacturers for the reflection magnitude of a high-reflective standard. Very probably, this has to do with the fact that the magnitudes of the reflection coefficients are assumed to be extremely close to the models. This in turn is a result of the physical law that the losses only contribute to a deviation from the ideal value of 1. Investigations on different offset shorts that have been carried out at METAS also seem to confirm that the attenuation of the offset line, which is a major cause for magnitude errors, is rather robust with respect to manufacturing parameters [4]. On the other hand, when considering connector effects and the fact that performance varies between manufacturers, there is no basis for a general statement of an uncertainty of only 0.001 for the real part of the residual reflection tracking coefficient, irrespective of frequency. Moreover, due to random errors, it isn't possible to verify figures of this order of magnitude.

Irrespective of an appropriate value, the contribution from the real part of residual reflection tracking to total uncertainty of the magnitude will indeed remain marginal, when the variances of the various contributions are added. So, a standard uncertainty of 0.0046 will be necessary to change a total standard uncertainty from 0.01 to 0.011.

When comparing eqs. (3) and (5), the uncertainty that can be attributed to residual source match will exceed that for residual reflection tracking. However this should not detract from the fact that actual residual source match ($\boldsymbol{\mu}$) may, by chance, be extremely lower than actual residual reflection tracking ($\boldsymbol{\tau}$). *Therefore, the uncertainty extracted from a ripple measurement of residual source match shall not be taken for residual reflection tracking.*

IV. Implications with using standard definitions from a calibration

When the (unknown) errors $\boldsymbol{\Delta s}$ and $\boldsymbol{\Delta o}$ are the outcome of a calibration rather than the final result of a dispersive manufacturing process, the issue of correlation between these quantities is more difficult to analyze. As long as no information is available on the calibration of the VNA that has been used to characterize both standards, nothing can be said on this. Thus, when transforming eq. (3) as to read uncertainties, the uncertainties associated to $\boldsymbol{\Delta s}$ and $\boldsymbol{\Delta o}$ are to be added linearly. An inequality sign has been chosen, however, to indicate that the calculated value will, generally, exceed the actual one:

$$u^2(\hat{\boldsymbol{\tau}}) < \frac{1}{4}\left[\frac{u(\hat{\boldsymbol{\Gamma}}_{\mathrm{s}})}{\hat{\boldsymbol{\Gamma}}_{\mathrm{s}}} + \frac{u(\hat{\boldsymbol{\Gamma}}_{\mathrm{o}})}{\hat{\boldsymbol{\Gamma}}_{\mathrm{o}}}\right]^2 + \frac{|\boldsymbol{\Delta x}|^2}{4}\left[u^2(\hat{\boldsymbol{\mu}}) + u^2(\hat{\boldsymbol{\delta}})\right] \tag{10}$$

$u(\hat{\boldsymbol{E}})$ represents a standard uncertainty of the expectation value of complex-valued quantity $\boldsymbol{E}$, and $\hat{\boldsymbol{\Gamma}}_{\mathrm{s}} = \boldsymbol{\Gamma}_{\mathrm{s}}^{\mathrm{c}}$, $\hat{\boldsymbol{\Gamma}}_{\mathrm{o}} = \boldsymbol{\Gamma}_{\mathrm{o}}^{\mathrm{c}}$, $\hat{\boldsymbol{\mu}} = \hat{\boldsymbol{\delta}} = \hat{\boldsymbol{\tau}} = 0$ represent best estimates. Even with poorly matched devices, e.g. for $|\boldsymbol{\Delta x}| = 0.15$, the second term only plays a minor role. When it shall be considered, the standard uncertainties of the residuals can be drawn from the ripple measurements, and $\boldsymbol{\Delta x}$ is available from the calibration reports of the high-reflective standards.

The uncertainties that are reported for the high-reflective calibration standards, are typically separated into phase uncertainties and uncertainties of the magnitude. The relative uncertainty of magnitude will mostly be somewhat lower than the quadrature component, i.e. the phase uncertainty in radians, but not to the extent as is expected with standard definitions. Therefore it is convenient to agree on only one value which can then be used in eq. (10). Considering that this equation regularly over-estimates actual uncertainty, one could make use of taking the square root of the mean variance of both components, rather than taking their maximum value.

Example: For a male offset open standard and a male offset short standard of the Keysight 85054B N calibration kit, a calibration certificate issued by METAS, makes the following statements for $\boldsymbol{S}_{11}$ at 18 GHz (in black):

	Mag	*U*(Mag)	*U*(Mag) / Mag	Phase	*U*(Phase)	
Open	0.9945	0.0045	0.0045	-103.6°	0.33°	0.0057 rad
Short	0.9939	0.0091	0.0092	82.3°	0.66°	0.0115 rad

From these values one gets (neglecting the term with $\boldsymbol{\Delta x}$): $\frac{u(\hat{\boldsymbol{\Gamma}}_{\mathrm{o}})}{\hat{\boldsymbol{\Gamma}}_{\mathrm{o}}} = 0.0026$ $\frac{u(\hat{\boldsymbol{\Gamma}}_{\mathrm{s}})}{\hat{\boldsymbol{\Gamma}}_{\mathrm{s}}} = 0.0052$ $u(\hat{\boldsymbol{\tau}}) = 0.004$

According to the derivation of eq. (10), uncertainties can also be calculated for residual source match and residual directivity, when eqs. (5) and (6) are used as a starting point. These values can then be compared with the measurement results that have been obtained for the ripple amplitudes, using the same standards. In the ideal case of no additional errors, i.e. with an ideal air line, no random errors, no drift and negligible connector effects, the ripple magnitude obtained for the measurement of residual directivity should be lower than 2.45 times the calibration uncertainty of the match. This is based on a coverage probability of 95 % and assumes a bivariate normal distribution of the errors of the match. For an (ideal) measurement of the source match ripple, smaller differences should be obtained due to the fact that eq. (10) tends to an over-estimation of uncertainty.

V. Conclusion

An approach has been presented, which allows to derive the uncertainty of the residual tracking parameter from the calibration data of the high-reflective standards within a regular SOLT calibration kit. Thus traceability can be established for measurements where the calibrated VNA has been characterized in terms of residual directivity and residual source match by the ripple method. In addition, it is shown how the residual tracking parameter can be drawn from the specifications of the manufacturer when model-based calibration standards are in use.

VI. References

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VII. Annex – Derivation of the residuals from the errors of the calibration standards

Since eq. (1) must be satisfied for all three calibration standards, three equations for the three unknowns ($\boldsymbol{\delta}$, $\boldsymbol{\mu}$, $\boldsymbol{\tau}$) are available. The usual approach of solving them comprises a linearization followed by a standardized solution of the linear system. Here, another approach is used, which is an extension of the idea behind Fig. 1, but on a more mathematical basis. This way, readers who are not so familiar with differentiating and solving linear equation systems are enabled to comprehend the computation and to establish a thorough understanding of the underlying principles.

Residual reflection tracking

Solving eq. (1) for $\boldsymbol{\tau}$ results in

$$\boldsymbol{\tau} = \frac{\boldsymbol{\Delta\Gamma}}{\boldsymbol{\Gamma}} - \frac{\boldsymbol{\delta}}{\boldsymbol{\Gamma}} - \boldsymbol{\mu\Gamma}\left(1 + \frac{\boldsymbol{\Delta\Gamma}}{\boldsymbol{\Gamma}}\right) + \boldsymbol{\mu\delta}\,. \tag{A1}$$

After introduction of the relative errors $\boldsymbol{\Delta s}$ and $\boldsymbol{\Delta o}$ and the congruence parameter $\boldsymbol{\Delta x}$ from eqs. (4) one gets the following equations for open and short:

$$\boldsymbol{\tau} = \boldsymbol{\Delta s} - \frac{\boldsymbol{\delta}}{\boldsymbol{\Gamma}_{\mathrm{s}}} - \boldsymbol{\mu\Gamma}_{\mathrm{s}}(1 + \boldsymbol{\Delta s}) + \boldsymbol{\mu\delta} \tag{A2}$$

$$\boldsymbol{\tau} = \boldsymbol{\Delta o} + \frac{\boldsymbol{\delta}}{\boldsymbol{\Gamma}_{\mathrm{s}}(1 + \boldsymbol{\Delta x})} + \boldsymbol{\mu\Gamma}_{\mathrm{s}}(1 + \boldsymbol{\Delta x})(1 + \boldsymbol{\Delta o}) + \boldsymbol{\mu\delta} \tag{A3}$$

Adding eqs. (A2) and (A3) yields

$$2\boldsymbol{\tau} = \boldsymbol{\Delta s} + \boldsymbol{\Delta o} - \frac{\boldsymbol{\delta\Delta x}}{\boldsymbol{\Gamma}_{\mathrm{s}}(1 + \boldsymbol{\Delta x})} + \boldsymbol{\mu\Gamma}_{\mathrm{s}}(\boldsymbol{\Delta x} + \boldsymbol{\Delta o} - \boldsymbol{\Delta s} + \boldsymbol{\Delta x\Delta o}) + 2\boldsymbol{\mu\delta} \tag{A4}$$

When neglecting second-order terms $\boldsymbol{\mu\Delta o}$, $\boldsymbol{\mu\Delta s}$, $\boldsymbol{\mu\delta}$ and all powers of $\boldsymbol{\Delta x}$, one finally gets

$$\boldsymbol{\tau} = \frac{\boldsymbol{\Delta s} + \boldsymbol{\Delta o}}{2} + \frac{\boldsymbol{\Delta x}}{2}\left(\boldsymbol{\mu\Gamma}_{\mathrm{s}} - \frac{\boldsymbol{\delta}}{\boldsymbol{\Gamma}_{\mathrm{s}}}\right) \tag*{(A5) / (3)}$$

Residual directivity

Solving eq. (1) for $\boldsymbol{\delta}$, one gets for the "Match"

$$\boldsymbol{\delta} = \boldsymbol{\Gamma}_{\mathrm{m}} + \boldsymbol{\Delta\Gamma}_{\mathrm{m}} - (1 + \boldsymbol{\tau})\frac{\boldsymbol{\Gamma}_{\mathrm{m}}}{1 - \boldsymbol{\mu\Gamma}_{\mathrm{m}}}\,, \tag{A6}$$

and after rearranging:

$$\boldsymbol{\delta} = \boldsymbol{\Delta\Gamma}_{\mathrm{m}} - \frac{1 + \boldsymbol{\tau} - 1 + \boldsymbol{\mu\Gamma}_{\mathrm{m}}}{1 - \boldsymbol{\mu\Gamma}_{\mathrm{m}}}\boldsymbol{\Gamma}_{\mathrm{m}} \tag{A7}$$

With $\boldsymbol{\Gamma}_{\mathrm{m}} \ll 1$, terms $\boldsymbol{\mu\Gamma}_{\mathrm{m}}$ can be neglected so that one finally gets

$$\boldsymbol{\delta} \approx \boldsymbol{\Delta\Gamma}_{\mathrm{m}} - \boldsymbol{\Gamma}_{\mathrm{m}}\boldsymbol{\tau} \tag*{(A8) / (6)}$$

Residual source match

Dividing eq. (1) by $\boldsymbol{\Gamma}$ results in

$$1 + \frac{\boldsymbol{\Delta\Gamma}}{\boldsymbol{\Gamma}} - \frac{\boldsymbol{\delta}}{\boldsymbol{\Gamma}} = \frac{1 + \boldsymbol{\tau}}{1 - \boldsymbol{\mu\Gamma}}\,. \tag{A9}$$

For the short standard and the open standard one gets with the definitions from eqs. (4):

$$1+\Delta s-\frac{\boldsymbol{\delta}}{\boldsymbol{\Gamma}_{\mathrm{s}}}=\frac{1+\boldsymbol{\tau}}{1-\boldsymbol{\mu}\boldsymbol{\Gamma}_{\mathrm{s}}} \tag{A10}$$

$$1+\Delta o+\frac{\boldsymbol{\delta}}{\boldsymbol{\Gamma}_{\mathrm{s}}(1+\boldsymbol{\Delta x})}=\frac{1+\boldsymbol{\tau}}{1+\boldsymbol{\mu}\boldsymbol{\Gamma}_{\mathrm{s}}(1+\boldsymbol{\Delta x})} \tag{A11}$$

Subtracting (A11) from (A10) yields

$$\Delta s-\Delta o-\frac{\boldsymbol{\delta}}{\boldsymbol{\Gamma}_{\mathrm{s}}}\cdot\frac{2+\boldsymbol{\Delta x}}{1+\boldsymbol{\Delta x}}=\boldsymbol{\mu}\boldsymbol{\Gamma}_{\mathrm{s}}(2+\boldsymbol{\Delta x})\frac{(1+\boldsymbol{\tau})}{(1-\boldsymbol{\mu}\boldsymbol{\Gamma}_{\mathrm{s}})(1+\boldsymbol{\mu}\boldsymbol{\Gamma}_{\mathrm{s}}(1+\boldsymbol{\Delta x}))}\,. \tag{A12}$$

When neglecting second-order terms of the residuals, one gets

$$\frac{\Delta s-\Delta o}{2}-\frac{\boldsymbol{\delta}}{\boldsymbol{\Gamma}_{\mathrm{s}}}\cdot\frac{1+\frac{\boldsymbol{\Delta x}}{2}}{1+\boldsymbol{\Delta x}}\approx\boldsymbol{\mu}\boldsymbol{\Gamma}_{\mathrm{s}}\left(1+\frac{\boldsymbol{\Delta x}}{2}\right), \tag{A13}$$

and after dividing the whole equation by $\left(1+\frac{\boldsymbol{\Delta x}}{2}\right)$:

$$\frac{\Delta s-\Delta o}{2}\left(1+\frac{\boldsymbol{\Delta x}}{2}\right)^{-1}-\frac{\boldsymbol{\delta}}{\boldsymbol{\Gamma}_{\mathrm{s}}}\cdot(1+\boldsymbol{\Delta x})^{-1}\approx\boldsymbol{\mu}\boldsymbol{\Gamma}_{\mathrm{s}}\,. \tag{A14}$$

A series expansion of $\left(1+\frac{\boldsymbol{\Delta x}}{2}\right)^{-1}$ and $(1+\boldsymbol{\Delta x})^{-1}$, including terms of 1st order only, yields:

$$\frac{\Delta s-\Delta o}{2}\left(1-\frac{\boldsymbol{\Delta x}}{2}\right)-\frac{\boldsymbol{\delta}}{\boldsymbol{\Gamma}_{\mathrm{s}}}\cdot(1-\boldsymbol{\Delta x})\approx\boldsymbol{\mu}\boldsymbol{\Gamma}_{\mathrm{s}}\,, \tag{A15}$$

Rearranging of terms leads to

$$\boldsymbol{\mu}\boldsymbol{\Gamma}_{\mathrm{s}}\approx\frac{\Delta s-\Delta o}{2}-\frac{\boldsymbol{\delta}}{\boldsymbol{\Gamma}_{\mathrm{s}}}-\frac{\boldsymbol{\Delta x}}{2}\left(\frac{\Delta s-\Delta o}{2}-2\frac{\boldsymbol{\delta}}{\boldsymbol{\Gamma}_{\mathrm{s}}}\right). \tag{A16}$$

Clearly, $\boldsymbol{\mu}\boldsymbol{\Gamma}_{\mathrm{s}}$ can roughly be approximated by the first and the second term of eq. (A16), ignoring the term with $\boldsymbol{\Delta x}$. Applying this approximation to the contents within the parentheses, and using eq. (A8) to substitute the residual directivity term (outside the parentheses), one finally gets

$$\boldsymbol{\mu}\approx\boldsymbol{\Gamma}_{\mathrm{s}}^{-1}\cdot\left[\frac{\Delta s-\Delta o}{2}-\frac{\Delta\boldsymbol{\Gamma}_{\mathrm{m}}}{\boldsymbol{\Gamma}_{\mathrm{s}}}-\frac{\boldsymbol{\Delta x}}{2}\left(\boldsymbol{\mu}\boldsymbol{\Gamma}_{\mathrm{s}}-\frac{\boldsymbol{\delta}}{\boldsymbol{\Gamma}_{\mathrm{s}}}\right)+\boldsymbol{\tau}\frac{\boldsymbol{\Gamma}_{\mathrm{m}}}{\boldsymbol{\Gamma}_{\mathrm{s}}}\right]. \tag{A17 / 5}$$